1/30/04
CD

EM 1438

Document And Report Documentation Page Submitted as edoc_1075487495

Report Documentation Page	Form Approved OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE **05 MAR 2003**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**	
4. TITLE AND SUBTITLE **Space-Time Codes for an Invariant Detector of Frequency-Hopped MIMO Communications**		5a. CONTRACT NUMBER	
		5b. GRANT NUMBER	
		5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)		5d. PROJECT NUMBER	
		5e. TASK NUMBER	
		5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **MIT Lincoln Laboratory, 244 Wood Street, Lexington, MA**		8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)	
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release, distribution unlimited**			
13. SUPPLEMENTARY NOTES **Also see: ADM001520 , The original document contains color images.**			
14. ABSTRACT			
15. SUBJECT TERMS			
16. SECURITY CLASSIFICATION OF:	17.	18.	19a. NAME OF RESPONSIBLE

| a. REPORT unclassified | b. ABSTRACT unclassified | c. THIS PAGE unclassified | LIMITATION OF ABSTRACT UU | NUMBER OF PAGES 20 | PERSON **Patricia Mawby, EM 1438** **PHONE:(703) 767-9038** **EMAIL:pmawby@dtic.mil** |

Standard Form 298 (Rev. 8-98) Prescribed by ANSI Std Z39-18
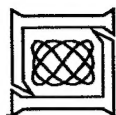
pwd: cannot determine current directory!

Space-Times Codes for an Invariant Detector of Frequency-Hopped MIMO Communications

Keith W. Forsythe [†]

[†]forsythe@ll.mit.edu
MIT Lincoln Laboratory
244 Wood Street, Lexington, MA

MIT Lincoln Laboratory

Codec Architecture for the Metachannel of an Invariant MIMO Detector

- Multiple input multiple output (MIMO) communications
  - Multiple transmitters coordinate channel coding by introducing space-time redundancy
  - Multiple receivers separate propagation modes in process of decoding
- Frequency-hopped MIMO
  - Channel transfer function (channel matrix) varies randomly hop-to-hop
  - Space-time coding occurs over hops and provides additional fading immunity and AJ
- Invariant detector
  - Short hops and low SNR can complicate channel estimation
  - Imposed detector invariances create metachannel robust to jamming and unknown channel
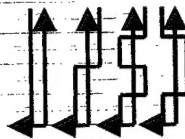
Belief network
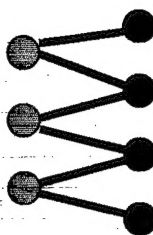
$$H = \begin{pmatrix} 1100 \\ 0110 \\ 0011 \end{pmatrix}$$
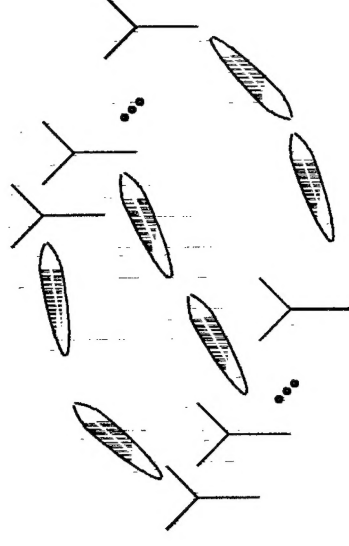
Walsh alphabet

Parity-check matrix

Channel matrix eigenvalues

Topics

- Introduction
- Signals in space
  - Signal model
  - Channel
  - Receiver
- Theoretical capacity
- Coding
  - Space-time inner codes
  - Low density parity-check outer codes
- Performance
  - Predictions
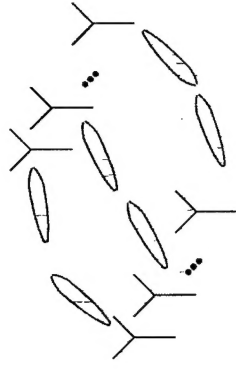 - Simulations
- Summary and Conclusions

Subspace Codes

- **Signal in additive noise (special case: # Rx = # Tx)**

$$\overset{n\times l}{\overbrace{Z}} = \overset{n\times n}{\underbrace{V}} \; \overset{n\times l}{\overbrace{S}} + \overset{n\times l}{\underbrace{N}}$$
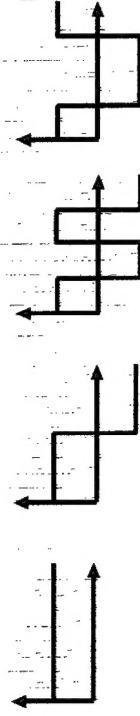
  Assume $l \geq 2n$

- **Motivation**

  – In absence of noise, rowspace(Z) = rowspace(S) for nonsingular V

  – Encode information bits in subspaces rowspace(S) and use only subspace of observations

  – Decision invariant to whitening transformations $Z \leftarrow R^{-1/2} Z$

- **Use scaled orthonormal signals ($SS^H \propto I_n$) to realize codes**

Invariant Detectors

- **Decision statistic** $D(Z, S)$

- **Invariances**

  - Subspace invariance

    $D(Z, S) = D(AZ, BS)$ for nonsingular A, B

  - Independence, with Gaussian samples

    $D(Z, S) = D(ZU, SU)$ for unitary U

- **Example:**

  $p(Z|R, V, S) = \pi^{-nl}|R|^{-l} \exp\{-\operatorname{tr}[(Z - VS)^H R^{-1}(Z - VS)]\}$

  $p(AZ|R, V, BS) = |AA^H|^{-l} p(Z|A^{-1}RA^{-H}, AVB, T)$

  $p(ZU|R, V, SU) = p(Z|R, V, S)$

  $D(Z, S) \stackrel{\triangle}{=} |ZZ^H|^l \cdot \max_{R,V} p(Z|R, V, S)$ has appropriate invariances

- **Maximal invariant** $D(Z, S)$ depends only on principal angles between subspaces rowspace(Z) and rowspace(S)

  - Other examples: $\operatorname{tr}(P_Z P_S)$, $|P_Z P_S|$, $\dfrac{|Z(I_l - P_S)Z^H|}{|ZZ^H|}$

Hopper Metachannel

- v varies randomly hop to hop
  - Prior on v: mean zero, complex, unity variance Gaussian i.i.d. entries

- Channel model
  - Transmit rowspace (S)
  - Receive rowspace (Z), with $Z = aVS + N$

- Maximum likelihood detector $\left(p \triangleq |a|^2\right)$

$$D(Z,S) = \left|I_n + \frac{p}{1+p}\,P_Z P_S\right|^{-l}$$

- Channel capacity

$$\mathbb{E}\left[\log_2\left((1+p)^{-l(l+1)/2}\left|I_n - \frac{p}{1+p}\,P_Z P_S\right|^{-l'}\right)/l\right]$$

- Suboptimal detector

$$D(Z,S) = e^{l\,\mathrm{tr}\,P_Z P_S}$$

Signal-to-Noise Ratios
Random Channel Matrices

- For m transmitters, n receivers, (average) data rate R, average element-to-element SNR, and bandwidth B, define $\frac{E_b}{N_0}$ to satisfy

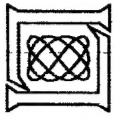$$mn\,\mathsf{SNR} = \frac{R}{B}\frac{E_b}{N_0}$$

- Motivating properties

$$\frac{E_b}{N_0} \longrightarrow \log(2) \text{ as } B \uparrow \infty$$

$$\log 2 \leq \frac{E_b}{N_0} \text{ using average rate } R$$

$$m, n \to \infty, \quad \frac{m}{n} \text{ fixed } \Rightarrow \log 2 = \frac{E_b}{N_0} \text{ for fixed rate } R$$

- Transmitted power proportional to $\frac{1}{n}\frac{E_b}{N_0}$

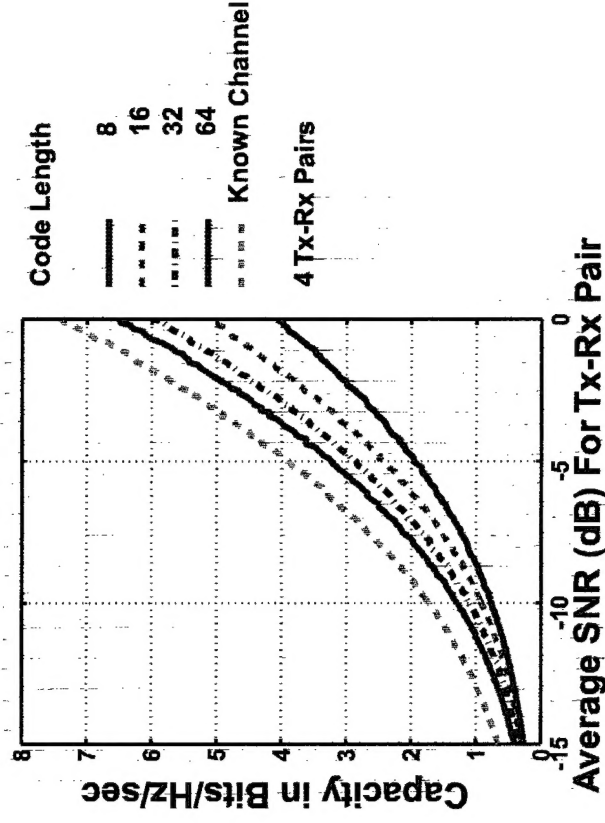 - MIMO $\frac{E_b}{N_0}$ is n times MISO $\frac{E_b}{N_0}$
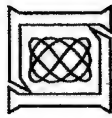
Capacity of the Metachannel

- **Upper bound on capacity**
  - Capacity when channel is tracked (known channel)

$$E_V \left[\log_2 \left(\left| I_n + |a|^2 VV^\dagger \right| \right) \right]$$

- **Performance**
  - As symbol length increases, capacity approaches that of tracked channel
  - Scaling all dimensions (number of receivers/transmitters and symbol length), channel behaves like infinite bandwidth channel but with added loss due to channel estimation.

## Capacity of 4X4 MIMO As a Function of Symbol Length

Code Length
8
16
32
64
Known Channel

4 Tx-Rx Pairs

Capacity in Bits/Hz/sec
Average SNR (dB) For Tx-Rx Pair

Space-Time Codes for the FH/PN Channel Concatenated Coding

- **Construct short space-time inner codes for each hop**
  - Invariant to channel matrix
  - Matrix symbols with 2^m values

- **Code over hops with low density parity-check (LDPC) outer code**
  - Length 1024, rate ½
  - 4 nonzero entries per column, 8 per row, totaling .8% of all entries
  - Symbols over GF(2^m)

- **Utilize invariant detector with probability vectors built from (quasi)-likelihoods**
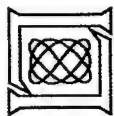
Locations of 4096 nonzero entries of 512 X 1024 paritycheck matrix
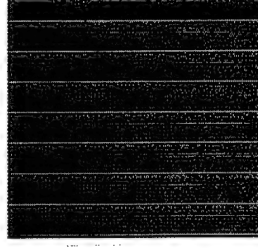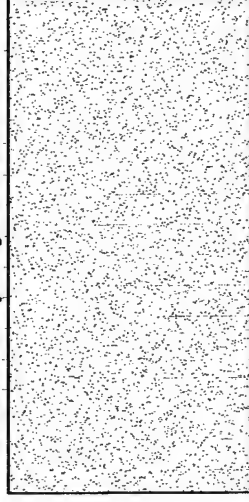
Nonzero entries of 1024 X 512 generator matrix

$$\left| I_n - \frac{p}{1+p} P_Z P_S \right|^{-l} e^{trl P_Z P_S}$$

MIT Lincoln Laboratory

Demodulating Matrix symbols

Probability vector

p_{m1} ... p_{mK}

Vector Norms

Correlators

Mth hop matrix

Z_m

Kth symbol value

Decision function: $e^{trlP_Z P_S}$
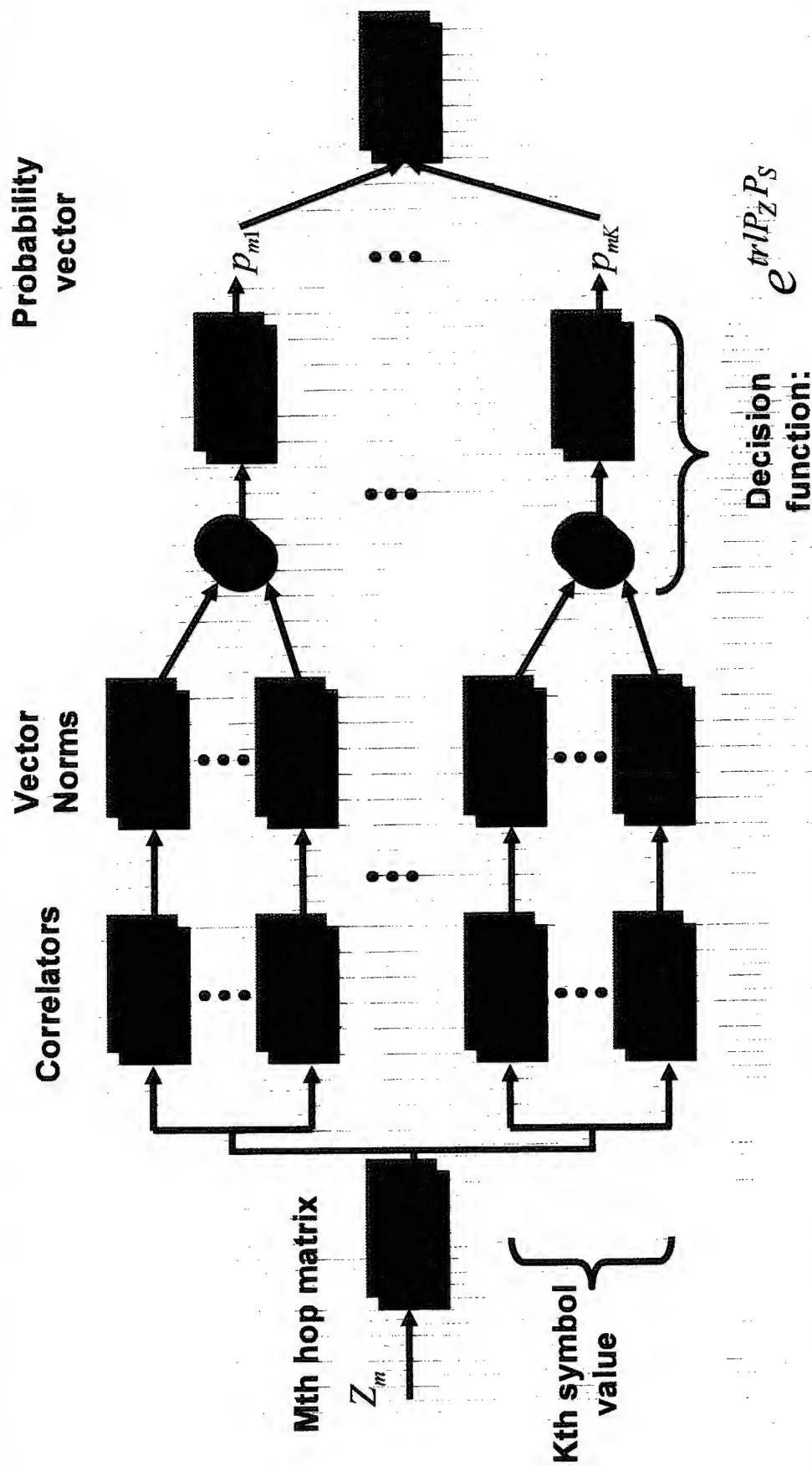
MIT Lincoln Laboratory
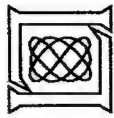
Decision Statistics For Matrix Symbols

- Quasi-likelihood and likelihood decision statistics provide similar performance
- Examples chosen from cases with about 5% symbol error probability
  - Histogram of components from length 16 probability vectors formed by (quasi)-likelihoods

Density of Suboptimal Quasi-likelihoods

$$e^{trlP_Z P_S}$$

Count vs QuasiLikelihood (dB)

Density of Likelihoods

$$\left| I_n - \frac{p}{1+p} P_Z P_S \right|^{-l}$$

Count vs Likelihood (dB)
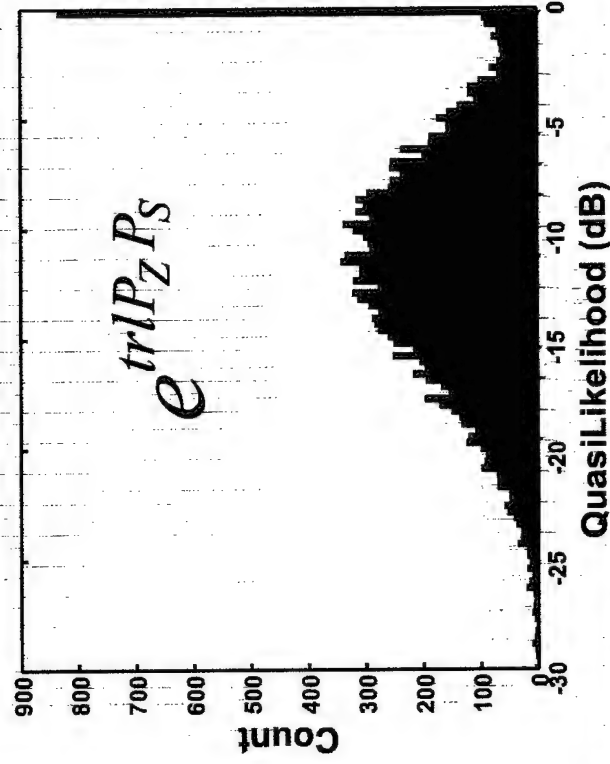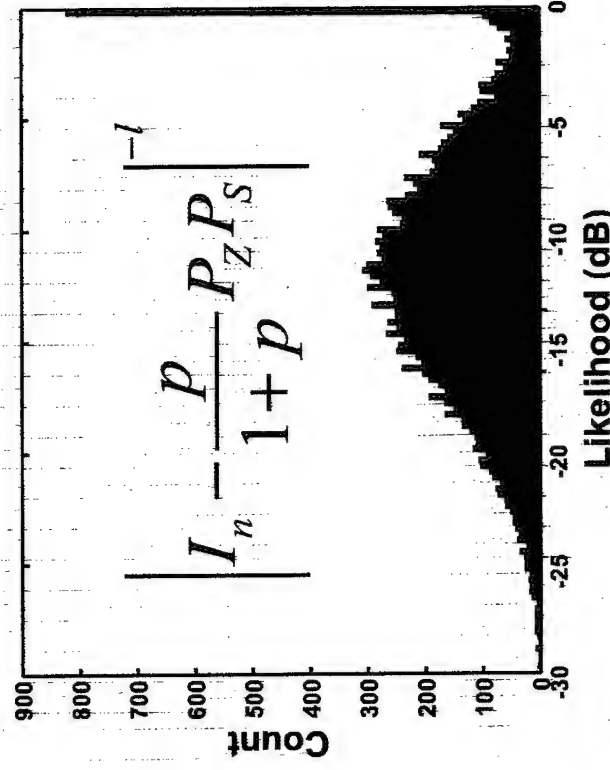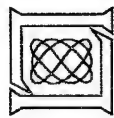
Graphical Decoding of Low Density Parity-Check Codes Using Bayesian Belief Networks

Variable dependencies

Loopless directed acyclic graph (DAG)
Directed Markov field
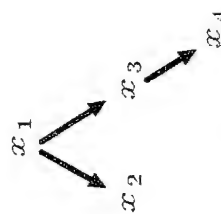Bayesian belief network

$$p(x_1, x_2, x_3, x_4) = p(x_4|x_3)p(x_3|x_1)p(x_2|x_1)p(x_1)$$

Message passing protocol

Messages passed	Received	Sent
likelihoods	λ^C	λ^P
priors	π^P	π^C
Belief		

Parent nodes (alphabets $u_k \in B_k$)

Update node (alphabet $x \in A$)

Child nodes
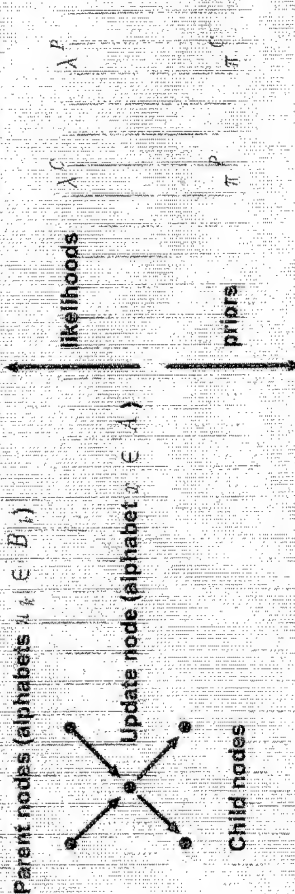
Node updates
Node calculations and messages

Network for a parity-check code
Bayesian Belief Network

Evidenciary nodes (observations)
Codeword components
Parity checks
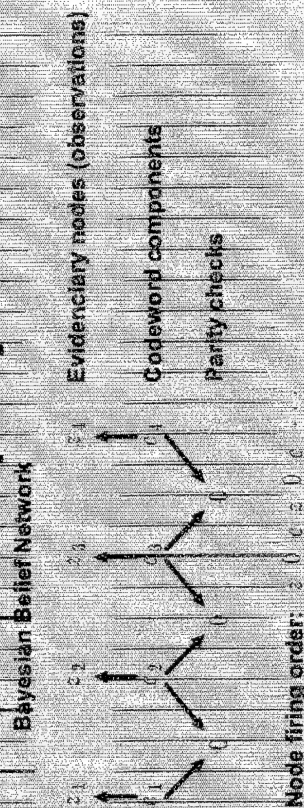
Node firing order:

Stopping rule: parity check satisfied

Parity check matrix

MIT Lincoln Laboratory

Constructions of Space-Time Inner Codes

Linear Block Codes

- Sets of orthonormal waveforms of length ι: $\{c_k\}$; $c_j \perp c_k$

- Matrix symbols $S(c)$

  $\phi_k : GF(2^k) \rightarrow c_k$, 1-1

  $c \in GF(2^k)^n$

  $$S(c) \triangleq = \begin{pmatrix} \phi_1(c_1) \\ \vdots \\ \phi_n(c_n) \end{pmatrix}$$

- Spectral efficiencies (r_s, r_t inner and outer code rates)

  $$\frac{R}{B} = r_t r_s \frac{k}{2^k}$$

Examples of Space-Time Inner Codes

Code	Parity Check Matrix	Field
(8,8,1)	0	$GF(2)$
(8,7,2)	$(1, 1, \ldots, 1)$	$GF(2)$
(8,6,3)	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \alpha & \alpha^2 & \cdots & \cdots & \alpha^6 \end{pmatrix}$	$GF(8)$
(8,4,4)	$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$	$GF(2)$
(8,3,6)	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & \alpha^2 & \alpha^3 & 1 \\ 0 & 0 & 1 & \alpha^4 & \alpha^4 & \alpha^4 & \alpha^6 & \alpha^3 \\ 0 & 1 & 0 & \alpha^8 & \alpha^8 & \cdots & \alpha^9 & \alpha^6 \\ 0 & 0 & 0 & 1 & \alpha^{12} & & & \end{pmatrix}$	$GF(8)$
(8,2,7)	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & \alpha & \alpha^2 & \alpha^2 & \alpha^2 \\ 0 & 0 & 1 & 0 & \alpha^3 & \alpha^3 & \alpha^4 & \alpha^4 \\ 0 & 0 & 0 & 1 & \alpha^5 & \alpha^5 & \alpha^6 & \alpha^6 \\ & & & & \alpha^{10} & \alpha^8 & \alpha^8 & \end{pmatrix}$	$GF(8)$
(8,1,8)	$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & \cdots & \cdots & 1 & \cdots & \cdots & 1 \end{pmatrix}$	$GF(2)$

Code	Parity Check Matrix	Field
(4,4,1)	0	$GF(2)$
(4,2,3)	$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & \alpha \end{pmatrix}$	$GF(4)$
(4,3,2)	$(1, 1, \ldots, 1)$	$GF(2)$
(4,1,4)	$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$	$GF(2)$

More of Space-Time Inner Codes

Steiner Systems

- **Orthonormal waveforms**

$$\{\vec{s}_k\}, \quad \vec{s}_j \perp \vec{s}_k, \quad j \neq k, \quad 1 \leq j, k \leq l$$

- **Matrix symbols**

$$E(c) \triangleq \begin{pmatrix} \vec{s}_{i_1} \\ \vdots \\ \vec{s}_{i_n} \end{pmatrix}$$

$$\{c_{i_1}, \ldots, c_{i_n}\} \text{ nonzero entries in } c, \; \mathrm{wt}(c) = n$$

- **Examples**

$$c = \begin{cases} (l = 16, 11, n = 4) & \textbf{140 codewords} \\ (l = 24, 12, n = 8) & \textbf{759 codewords} \end{cases} \quad \mathrm{wt}(c) = n$$

- **Subspace separations**

$$\dim(E(c) \cap E(c')) \leq \begin{cases} 2 & (16, 11, 4) \\ 4 & (24, 12, 8) \end{cases} \quad c \neq c'$$

Maximally separated away from intersection

Theoretical Predictions

Approximate Error Exponents

- **Effective SNR (interference covariance R_I as r.v. hop to hop)**

$$\frac{n\,d}{4}\,\left(\frac{\mathrm{tr}(\mathbb{E}[R_I^{-1}VV^H])}{n^2}\right)^2 \cdot (i\mathrm{SNR})^2$$

- **Bounds for linear block codes ($D/N \leq 1/2$)**

  Gilbert-Varshamov (GS):
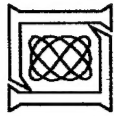 $$\sum_{k=0}^{D-2}(q-1)^k\binom{N-1}{k} < q^\tau$$

  Rank: $D \leq N - K + 1$

- **Asymptotic form of Gilbert-Varshamov bound**

$$G_q(x) \stackrel{\triangle}{=} \log q - x\log(q-1) - x\log x - (1-x)\log(1-x)$$
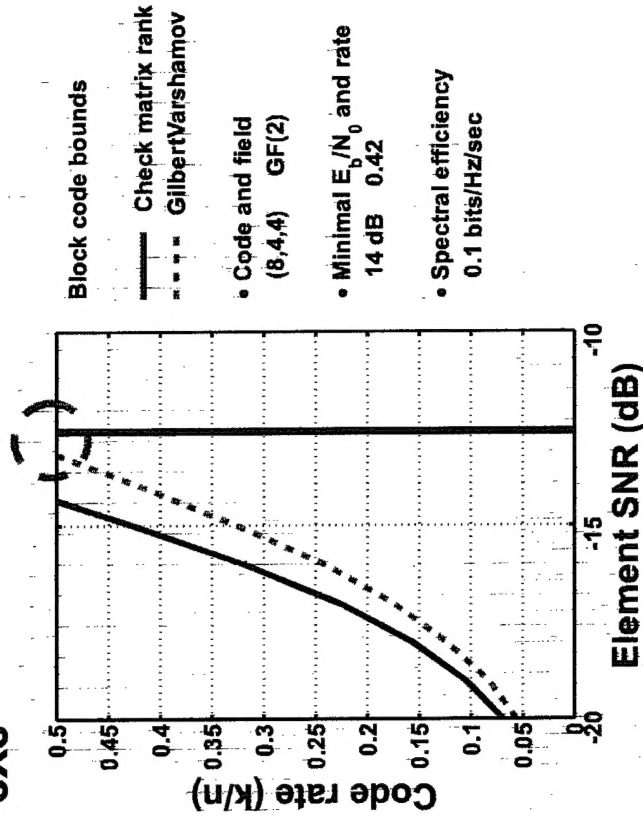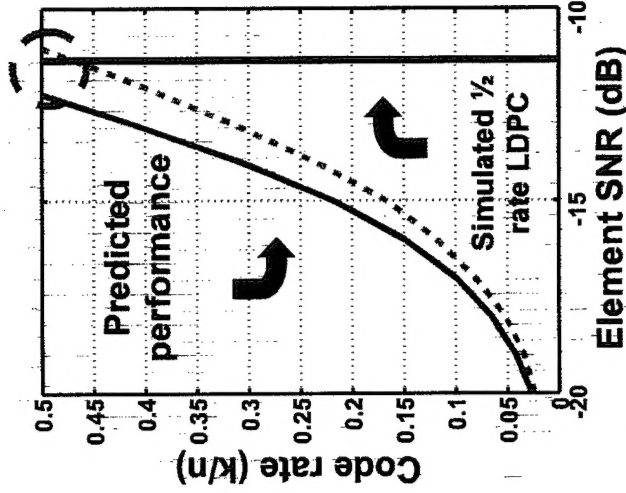
$$K/N\log q = G_q(D/N)$$

- **Error exponent (GS):**
$$\frac{K}{N}\log q - \mathrm{SNR}_{\mathrm{eff}}\,G_q^{-1}\left(\frac{K}{N}\log q\right)$$

Comparison of Theoretical and Simulated Performance

- Predicted performance expresses code rate in terms of SNR

- Minimizing $\frac{E_b}{N_0}$ over SNR results in optimal codes of rate near 1/2

- Predicted performance agrees closely with simulated ½ rate LDPC outer code concatenated with space-time inner codes

4X4 ⟵ MIMO ⟶ 8X8

Block code bounds
—— Rank
‑ ‑ ‑ Gilbert-Varshamov

- Code and field
  (4,2,3)   GF(4)

- Minimal E_b/N_0 and rate
  10 dB   0.4

- Spectral efficiency
  0.1 bits/Hz/sec

Block code bounds
—— Check matrix rank
‑ ‑ ‑ Gilbert-Varshamov

- Code and field
  (8,4,4)   GF(2)

- Minimal E_b/N_0 and rate
  14 dB   0.42

- Spectral efficiency
  0.1 bits/Hz/sec

Predicted performance

Simulated ½ rate LDPC

Code rate (k/n)
Element SNR (dB)

Code rate (k/n)
Element SNR (dB)

MIT Lincoln Laboratory

Simulated Performance With Jamming and Nonrandom Channel Matrices
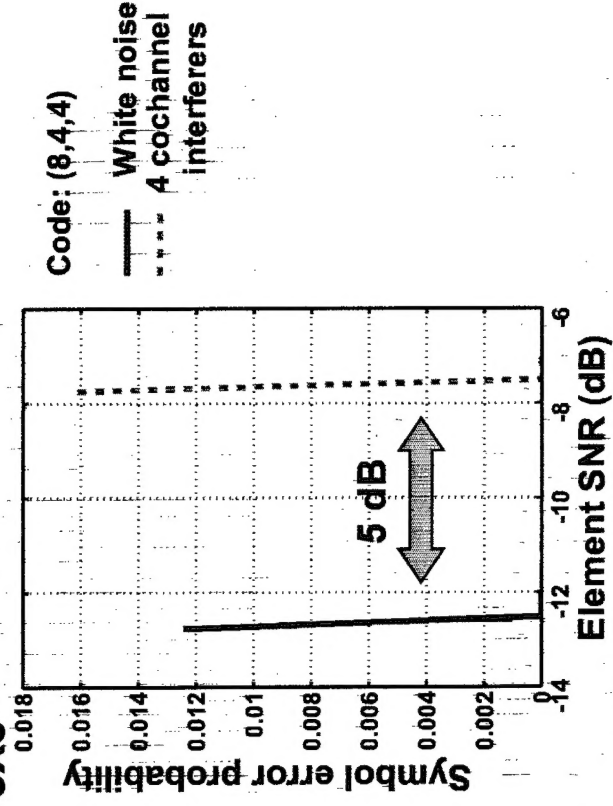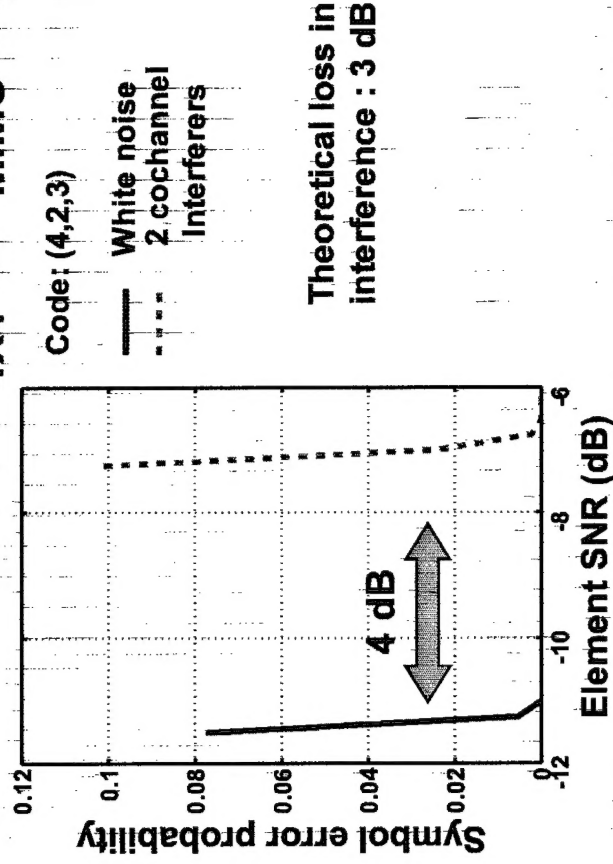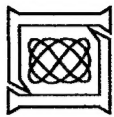
- **Theoretically, K jammers result in (N-K)/N SINR loss**

- **Simulated results indicate losses are somewhat higher**

- **When channel matrix is constant over all hops, predicted performance agrees with random variation provided received power is scaled to make $tr(VV^{\dagger})/n^2$ unity**

Simulated performance with and without jamming

$$4X4 \longrightarrow MIMO \longrightarrow 8X8$$

Code: (4,2,3)

——— White noise

- - - - 2 cochannel interferers

Theoretical loss in interference : 3 dB

4 dB

Element SNR (dB)

Symbol error probability
0.12, 0.1, 0.08, 0.06, 0.04, 0.02
-12, -10, -8, -6

Code: (8,4,4)

——— White noise

- - - - 4 cochannel interferers

5 dB

Element SNR (dB)

Symbol error probability
0.018, 0.016, 0.014, 0.012, 0.01, 0.008, 0.006, 0.004, 0.002
-14, -12, -10, -8, -6

MIT Lincoln Laboratory
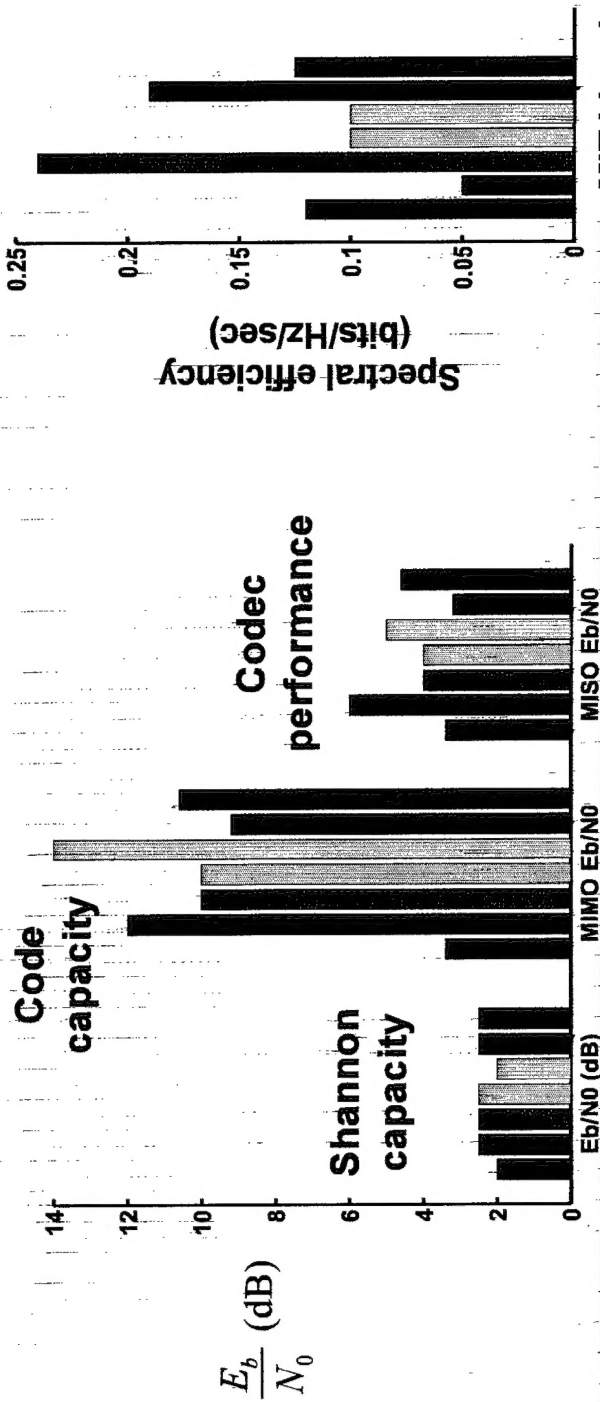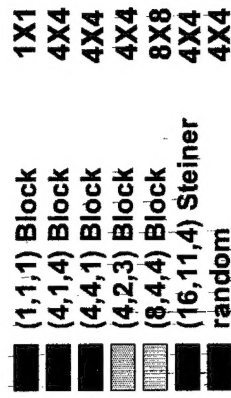
Summary of Performance
Random Channel Matrices

- **Codes**
  - Inner code specified by block code parameters, Steiner system parameters or random matrix symbols (4X4 MIMO with 16 length 16 matrix symbols)
  - Outer code: (1024,512) LDPC over GF(16), GF(128), or GF(256)

- **Performance**
  - Predicted by effective SNR and Gilbert-Varshamov bounds (except random case)
  - Bounds validated by simulation (within several tenths dB)

(1,1,1) Block	1X1
(4,1,4) Block	4X4
(4,4,1) Block	4X4
(4,2,3) Block	4X4
(8,4,4) Block	8X8
(16,11,4) Steiner	4X4
random	4X4

Spectral efficiency (bits/Hz/sec)

$\frac{E_b}{N_0}$ (dB)

Code capacity

Codec performance

Shannon capacity

MIMO Eb/N0

MISO Eb/N0

Eb/N0 (dB)

Summary and Conclusions

- **Class of invariant detectors formulated for robust demodulation and decoding in unknown interference with unknown channels**

  - Capacity evaluated for the frequency-hopped (FH) channel as received by an invariant detector

- **Family of concatenated codes examined for frequency-hopped, pseudo-noise (FH/PN) channel**

  - Family uses linear block codes, Steiner systems, etc. for space-time inner code matrix symbols and low density parity-check outer codes

  - Theoretical performance agrees with simulations

- **Performance**

  - Concatenated codes considered operate around 3 to 4 dB (MISO) $\frac{E_b}{N_o}$

  - Concatenated codes examined are 7-8 dB worse than channel capacity bound in white noise

  - Space-time codes provide n^2 diversity even when channel matrices remain constant hop to hop

  - Space-time codes and invariant detector handle interferers and unknown channels gracefully with little sensitivity to interference geometry

MIT Lincoln Laboratory